B.Sc. Hons. Part II

Vector calculus

1. Find ∇r^m

Sol^n $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

$\Rightarrow r^2 = x^2 + y^2 + z^2 \Rightarrow r = \sqrt{x^2+y^2+z^2}$

$\Rightarrow r^m = (x^2+y^2+z^2)^{m/2}$

$\therefore \nabla r^m = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)(x^2+y^2+z^2)^{m/2}$

$= \vec{i}\frac{\partial}{\partial x}(x^2+y^2+z^2)^{m/2} + \vec{j}\frac{\partial}{\partial y}(x^2+y^2+z^2)^{m/2}$

$+ \vec{k}\frac{\partial}{\partial z}(x^2+y^2+z^2)^{m/2}$

$\Rightarrow \nabla r^m = \vec{i}\frac{m}{2}(x^2+y^2+z^2)^{m/2-1}\cdot 2x + \vec{j}\frac{m}{2}(x^2+y^2+z^2)^{\frac{m}{2}-1}\cdot 2y$

$+ \vec{k}\frac{m}{2}(x^2+y^2+z^2)^{\frac{m}{2}-1}\cdot 2z$

$\Rightarrow \nabla r^m = \frac{m}{2}\cdot(x^2+y^2+z^2)^{\frac{m-2}{2}}\left[2x\vec{i} + 2y\vec{j} + 2z\vec{k}\right]$

$= \frac{m}{2}\cdot(r^2)^{\frac{m-2}{2}}\cdot \times 2\vec{r}$

$$\Rightarrow \boxed{\nabla r^n = n r^{n-2} \vec{r}}$$

If we put $n = 1$ then the above relation results into

$$\boxed{\nabla r = \frac{n}{r} \vec{r} = \frac{1}{r} \vec{r}}$$

Put $n = 2$ then

$$\boxed{\nabla r^2 = 2 \vec{r}}$$

Put $n = -1$ then

$$\nabla \frac{1}{r} = (-1) r^{-3} \vec{r}$$

i.e. $$\boxed{\nabla \left(\frac{1}{r}\right) = \frac{-1}{r^3} \vec{r}}$$